Heat Conductivity of Monatomic Gases

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In Ref. 1, the author has described the application of the principle of corresponding states to the viscosity of gases at atmospheric pressure. Below the same principle is applied to the heat conductivity of monatomic gases. The Chapman-Enskog theory, on whose results we base our choice of a system of variables, is strictly applicable only to monatomic gases. The discussion will be confined exclusively to gases of this type.

At low densities the Chapman-Enskog theory gives the following expression[2] for the heat conductivity of monatomic gases

$$\lambda = (T/M)^{1/2} r_0^{-2} F(kT/\epsilon) \tag{1}$$

where k is Boltzmann's constant; r_0 and ϵ are, respectively, a characteristic dimension (distance) and energy, determined by the properties of the molecule.

It is known[2] that ϵ/k and $r_0^3 N$ (N = Avogadro's number) are proportional to T_{cr} and V_{cr}. Then from Eq. (1), in accordance with the principle of corresponding states, we find that the reduced heat conductivity of monatomic gases $\lambda_{red} = \lambda V_{cr}^{2/3} (M/T)^{1/2}$ is a function only of the reduced temperature τ:

$$\lambda_{red} = f(\tau) \tag{2}$$

Generalizing the reduced heat conductivity λ_{red}, rather than $\lambda_{red}' = \lambda/\lambda_{1cr}$, as favored by other authors,[3, 4] is advantageous because in our case it is not essential to possess experimental data in the vicinity of T_{cr} for the determination of λ_{1cr}.

As Fig. 1 shows, the experimental points, obtained by different methods and embracing a wide range of variation in temperature (from −183° to 1100°C), when plotted in terms of λ_{red} vs τ, lie, with relatively unimportant scatter, on a single curve. The mean deviation is 4.5%. The slight scatter is due mainly to discrepancies among the data supplied by different authors. Thus, for example, L. S. Zaitseva's[5] experimental points for argon, krypton, xenon, and neon can be described by a single curve with a mean scatter of only 1%, whereas the discrepancy between the data of Zaitseva and Kannuluik and Carman[6] reaches 3–7%. As with the viscosity,[1] owing to the influence of quantum effects, helium constitutes an exception (in view of the considerable difference between helium and the other gases, data on the former's heat conductivity have been omitted from Fig. 1).

In the region τ = 0.8–8 the generalized curve in Fig. 1 may be described with sufficient accuracy (±1%) by the following expression in the form of a Sutherland equation:

$$\lambda_{red} = (4.24 \cdot 10^{-6})/(1 + 1.00/\tau) \tag{3}$$

where λ is measured in cal/cm·sec·deg, V_{cr} in cm³/g-mole, and T in °K.

The curve in Fig. 1 can be used to determine the heat conductivities of monatomic gases (except for helium) within the range τ = 0.5–14.

It would be useful to solve the problem of the temperature dependence of the proportionality factor ϵ, entering into the known relation, obtained from the molecular-kinetic theory of gases:

$$\lambda = \epsilon c_v \eta \tag{4}$$

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For elastic spherical molecules the theory requires that ϵ should be independent of the temperature. According to Enskog, however, when the intermolecular forces of attraction are taken into account, there is a slight variation in ϵ with temperature:

$$\epsilon = 2.522/(1 + 0.038\, C/T) \tag{5}$$

where C is Sutherland's constant.

Aiken's relation $\epsilon = (9K - 5)/4$, where $K = c_p/c_v$, indicates that the coefficient ϵ for monatomic gases is independent of temperature. Keyes[7] did not succeed in establishing a definite law of variation of ϵ. Zaitseva's work[5] shows that ϵ increases both with increase in the molecular weight and with increase in temperature (but much more strongly than reported by Enskog) in accordance with an expression analogous to (5) (when $t > 0$°C).

The results of this and the author's previous paper[1] make it possible to evaluate the dependence of the quantity ϵ for monatomic gases on the reduced temperature in generalized form.

In our case the ratio of the reduced heat conductivity and viscosity is:

$$\frac{\lambda_{red}}{\eta_{red}} = [\lambda V_{cr}^{2/3}(M/T)^{1/2}]:[\eta V^{2/3}(MT)^{-1/2}] = \frac{\lambda}{\eta} M \tag{6}$$

From Eqs. (4) and (6), taking into account that for monatomic gases $c_v = 2.98/M$ (cal/g·deg), we have

$$\epsilon = (1/2.98)(\lambda_{red}/\eta_{red}) \tag{7}$$

or

$$\epsilon = f'(\tau) \tag{8}$$

that is, within the limits of accuracy of the generalization of heat conductivity and viscosity (±2%) over a broad range of variation of τ (0.5–14), the coefficient ϵ for monatomic gases (excepting helium) depends only on the reduced temperature.

For the viscosity of monatomic gases the dependence of η_{red} on τ has the form[1]

$$\eta_{red} = (5.35 \cdot 10^{-7})/(1 + 0.88/\tau) \tag{9}$$

From Eqs. (3), (8), and (9) we obtain the relation for the computation of ϵ as a function of temperature (τ = 0.8–8.0):

$$\epsilon = 2.65[(\tau + 0.88)/(\tau + 1.00)] \tag{10}$$

In Fig. 2, curve 2 corresponds to Eq. (10). It is clear that the value of ϵ for monatomic gases increases with temperature, the rate of increase declining with increase in τ, whereas when $\tau \to \infty$ the coefficient ϵ tends to a constant value equal to 2.65. Curve 2 in Fig. 2 can also be described, correct to ±1%, by an equation of the Enskog type:

$$\epsilon = 2.65/(1 + 0.07/\tau) \tag{11}$$

In its turn Enskog's Eq. (5) can be expressed in terms of the reduced quantities. For heavy monatomic gases (Ar, Kr, Xe) Sutherland's reduced constant $C_{red} = C/T_{cr}$ can be taken as equal to 0.88. Then Eq. (5) will have the form

$$\epsilon = 2.522/(1 + 0.0334/\tau) \tag{12}$$

Curve 1 in Fig. 2 corresponds to Enskog's modified equation (12). On comparing (11) and (12) we find that the dependence of the coefficient on temperature, obtained on the

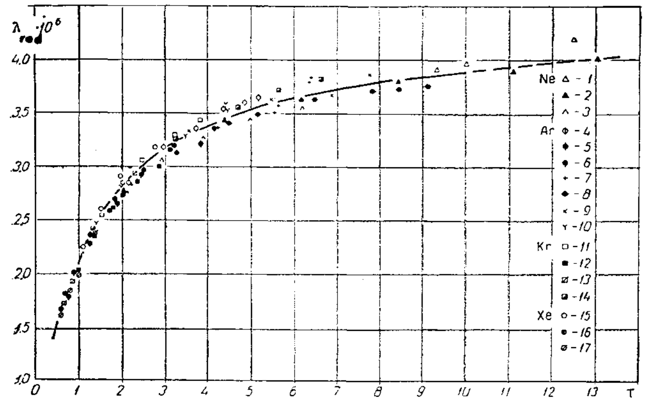

Fig. 1 Generalization of heat conductivity of monatomic gases according to data of: 1, 4, 11, 15—Zaitseva[5]; 2, 5, 12, 16—Kannuluik and Carman[6]; 3, 6, 13, 17—Keyes[7]; 7—Rothman and Bromley[10]; 8, 14—Schäfer and Reiter[8]; 9—Vines[9]; 10—Tsederberg, Popov, and Morozova[11]

basis of our generalization, is roughly twice that given by Enskog's equation.

In conclusion we should point out that the equation

$$\lambda = \lambda_0(T/T_0)^n \quad (13)$$

(where λ_0 is the heat conductivity for $T_0 = 273°K$, and n is an exponent depending on the nature of the gas), which is at present widely used for extrapolation, is obviously less accurate than expression (4) or an equation of the Sutherland type. The latter give a qualitatively superior representation of the dependence of the heat conductivity on temperature. Thus, for example, from an analysis of experimental material (up to 515°C) for argon, Zaitseva[5] has found $\lambda_0 = 142$ kcal/m·hr·deg and $n = 0.80$. Then Eq. (13), with $t = 1000°C$, gives $\lambda = 486$ kcal/m·hr·deg, but (3) and (4) give only 433 and 443 kcal/m·hr·deg, respectively. Here it is assumed that $\eta = 616 \cdot 10^{-6}$ g/cm·sec, according to experimental data, and $\epsilon = 2.68$, according to an equation of type (5). As shown by the experimental data (Fig. 1), for $t = 1000°C$, the heat conductivity of argon is equal to 437 kcal/m·hr·deg—i.e., extrapolation from Eq. (13) leads to an error of ~10%, whereas Eqs. (3) and (4) give good agreement with experiment. An analogous result is obtained for krypton, which has been investigated experimentally up to a temperature of 1100°C.[8]

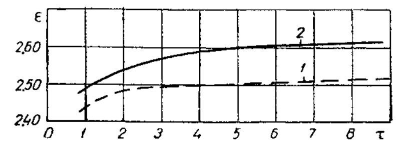

Fig. 2 Coefficient ϵ as a function of the reduced temperature: 1) according to the modified Enskog equation (12); 2) according to the results of the author's generalization

References

[1] Tarzimanov, A. A., Inzh.-Fiz. Zh. (Eng.-Phys. J.), no. 2 (1960).

[2] Hirschfelder, J. O., Curtis, C. F., and Bird, R. B., *Molecular Theory of Gases and Liquids* (John Wiley & Sons, Inc., New York, 1954).

[3] Owens, E. J. and Thodos, G., J. Am. Inst. Chem. Engrs. **3**, 454 (1957).

[4] Tsederberg, N. V., Teploenerget. (Thermal Power Eng.), no. 7 (1956).

[5] Zaitseva, L. S., Dissertation, MAI (1956); Zh. Tekhn. Fiz. (J. Theoret. Phys.), no. 4 (1959).

[6] Kannuluik, W. G. and Carman, E. U., Proc. Phys. Soc. **65B**, 701 (1952).

[7] Keyes, F. G., Trans. Am. Soc. Mech. Engrs. **76,** 809 (1954); **77,** 1395 (1955).

[8] Schäfer, K. L. and Reiter, F. W., Z. Elektrochem. **61,** 1230 (1957).

[9] Vines, R. G., Trans. Am. Soc. Mech. Engrs., Ser. C: no. 1 (1960).

[10] Rothman, A. I. and Bromley, L. A., Ind. Eng. Chem. **47,** 899 (1955).

[11] Tsederberg, N. V., Popov, V. N., and Morozova, N. A., Teploenerget., no. 6 (1960).

[12] *Thermophysical Properties of Materials,* handbook edited by N. B. Vargaftik (Gosenergoizdat, 1956).

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Comment on "Equations of the Precessional Theory of Gyroscopes"

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Nomenclature

A,B,C = moments of inertia of a rigid body about its principal axes a,b,c
ω = component of angular velocity of a rigid body
Ω = component of angular velocity of a system of orthogonal axes x,y,z
M = component of sum of the moments of forces applied to a rigid body
f = frequency

Subscripts

a,b,c = principal axes of inertia of a rigid body
x,y,z = system of orthogonal axes; axis z is identical with axis c

In a translation of a paper by L. I. Kuznetsov,† three equations are given which, when expressed in symbols more familiar to Western readers, take the form

$$\begin{aligned} M_x &= A(\dot{\Omega}_x - \Omega_z\Omega_y) + C\omega_z\Omega_y \\ M_y &= A(\dot{\Omega}_y + \Omega_z\Omega_x) - C\omega_z\Omega_x \\ M_z &= C\dot{\omega}_z \end{aligned} \quad (1)$$

These are closely related to Euler's equations but are not usually so called.

The author then introduces approximate equations which may be called the equations of precession. In the nomenclature used here these equations are

$$M_x = C\omega_z\Omega_y \qquad M_y = -C\omega_z\Omega_x \qquad M_z = C\dot{\omega}_z \quad (2)$$

Euler's equations are valid for any rigid body. The body need not be dynamically symmetrical about any axis. All vectors are referred to a set of orthogonal axes a,b,c which are fixed in the body and which coincide with the principal axes of inertia.

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† Kuznetsov, L. I., "Equations of the precessional theory of gyroscopes," AIAA J. 1, 271–274 (1963).